The Sunyaev-Zeldovich Effect

[Sunyaev & Zeldovich 1969, *Nature,* **223,** 721]
[Carlstrom *et al.* 2002, *ARAA,* **40,** 643]

There are four ways to get the distances to extragalactic objects that do not depend in any way on the distance ladder. The first is through simple geometry, as in the light echo of SN 1987A or the proper motions of the cirumnuclear masers of NGC 4258. (The proper motions of masers have also been used to derive a geometric distance to a few, more distant galaxies.) The second is the Expanding Photospheres Method for Type II supernovae. (This is similar to Baade-Wesselink.) The third relies on the time delays of a gravitational lens. The fourth is the Sunyaev-Zeldovich (SZ) effect.

The SZ effect is caused by microwave photons scattering off of the hot x-ray gas in clusters. To see how this works, we must first consider the scattering of low-energy photons off high-energy electrons. If the electrons are at rest, this is Compton scattering; if the electrons are moving, it's called Inverse Compton. The equation for inverse Compton is relatively simple in the case of $h\nu \ll kT_e$ and $h\nu \gg kT_e$, but for the intermediate cases, it's given by the Kompaneets Equation. First, let y be the (temperature weighted) density of electrons that the photons must pass through on their way to us. In other words,

$$y = \frac{k\sigma_T}{m_e c^2} \int n_e T_e dl \tag{20.01}$$

where σ_T is the Thomson cross section of the electron, n_e is the electron density, T_e is the electron temperature, and l is the path length. If we further let T be the (blackbody) temperature of the photons, and x be the ratio of photon energy to electron energy,

i.e.,

$$x = \frac{h\nu}{kT_e} \tag{20.02}$$

then the Kompaneets Equation is

$$\frac{dN}{dy} = \frac{1}{x^2}\frac{d}{dx}\left\{x^4\left(\frac{dN}{dx} + N + N^2\right)\right\} \tag{20.03}$$

where N is the photon occupation number

$$N = \frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{20.04}$$

For simplicity, let's assign

$$z = \frac{h\nu}{kT} = \frac{T_e}{T}x \tag{20.05}$$

We can then write the Kompaneets equation as

$$\frac{dN}{dy} = \frac{1}{x^2}\frac{d}{dx}\left\{x^4\left(\frac{T_e}{T}\frac{dN}{dz} + N + N^2\right)\right\} \tag{20.06}$$

Now let's examine the 3 terms in the parenthesis.

$$N = (e^z - 1)^{-1}; \quad N^2 = (e^z - 1)^{-2}; \qquad \frac{dN}{dz} = -e^z\left(e^z - 1\right)^{-2} \tag{20.07}$$

Since $z > 0$, $e^z > 1$, and $\left|\frac{dN}{dz}\right| > N^2$. Also, since $e^z > e^z - 1$, $\left|\frac{dN}{dz}\right| > N$. So the first term dominates, and

$$\frac{dN}{dy} \approx \frac{1}{x^2}\frac{d}{dx}\left\{x^4\frac{T_e}{T}\frac{dN}{dz}\right\} = \frac{1}{x^2}\frac{d}{dx}\left\{x^4\frac{dN}{dx}\right\} \tag{20.08}$$

Now let $a = T_e/T$, so that $N = (e^{ax} - 1)^{-1}$. For microwave photons, $h\nu \ll kT_e$, which means $x \ll 1$, so we can Taylor expand e^{ax} to get

$$N = (e^{ax} - 1)^{-1} \approx (1 + ax + \ldots - 1)^{-1} = \frac{1}{ax} \tag{20.09}$$

This implies that

$$\frac{dN}{dx} = \frac{d}{dx}\left(\frac{1}{ax}\right) = -\frac{1}{ax^2} \tag{20.10}$$

and

$$\frac{d}{dx}\left\{x^4 \frac{dN}{dx}\right\} = \frac{d}{dx}\left\{-x^4 \frac{1}{ax^2}\right\} = -\frac{d}{dx}\left\{-\frac{x^2}{a}\right\} = -2\frac{x}{a} \tag{20.11}$$

So the Kompaneets equation greatly simplifies to

$$\frac{dN}{dy} = \frac{1}{x^2}\frac{d}{dx}\left\{x^4 \frac{dN}{dx}\right\} \frac{1}{x^2}\left(-\frac{2x}{a}\right) = -\frac{2}{ax} = -2N \tag{20.12}$$

or

$$\frac{dN}{dy} = -2N \tag{20.13}$$

Now let's solve this equation

$$\int -2dy = \int \frac{dN}{N} \implies \quad = -2y = \ln\left(\frac{N'}{N}\right) \tag{20.14}$$

where N' is the photon occupation after passing through the cluster. Since the observed amount of this distortion is very small, we can Taylor expand $\ln N$ to get

$$-2y = \ln\left(\frac{N + dN}{N}\right) = \ln\left(1 + \frac{dN}{N}\right) \approx \frac{dN}{N} \tag{20.15}$$

or, since

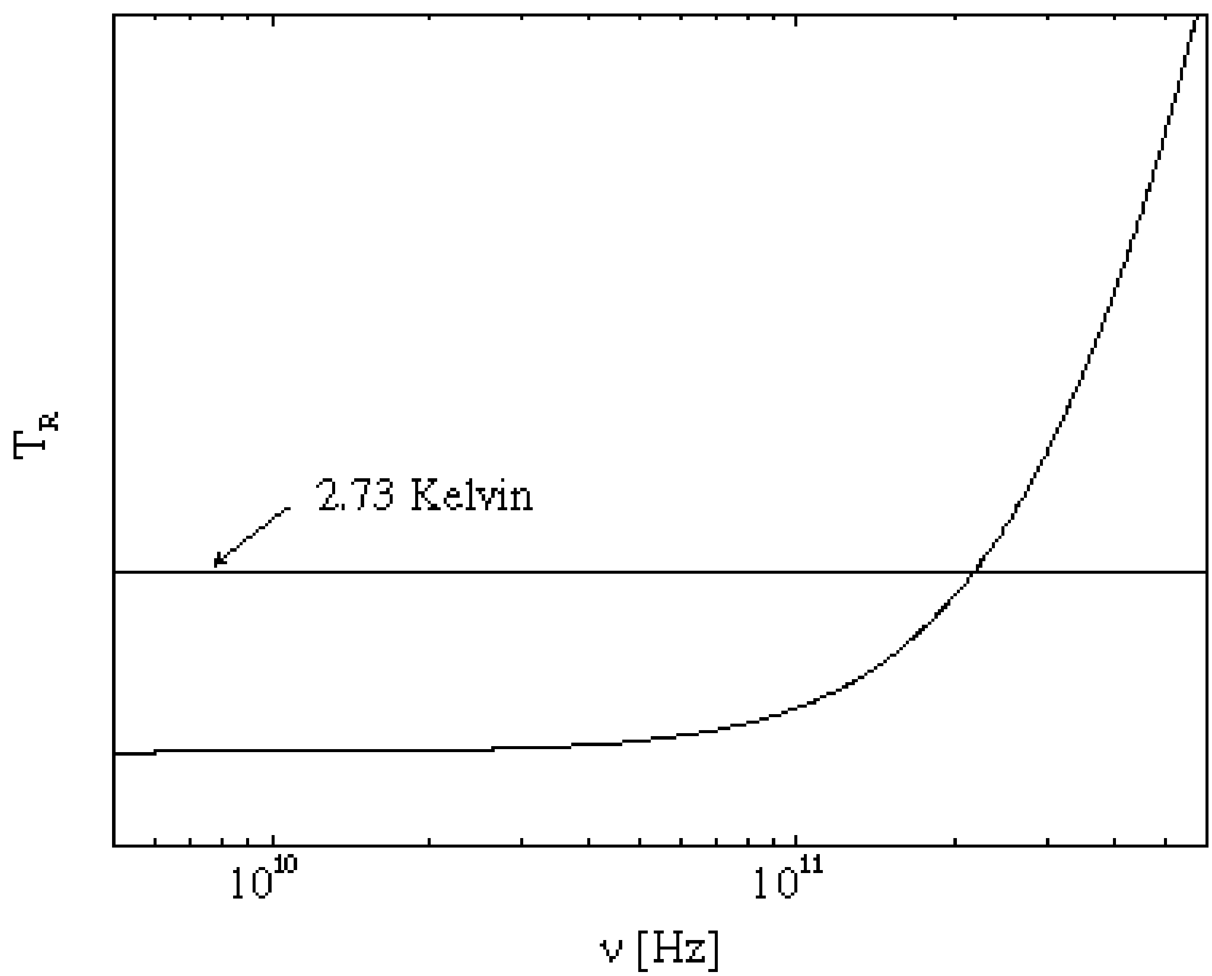
$$N = \frac{1}{e^x - 1} \approx \frac{1}{x} = \frac{kT}{h\nu} \tag{20.16}$$

we have

$$\frac{dN}{N} \approx \frac{dT}{T} = -2y \tag{20.17}$$

In other words, photons will be upscattered out of the microwave to high energies. That will cause the microwave region of the spectrum to appear fainter, or equivalently, to appear to have a lower temperature. Specifically

$$\frac{\delta T}{T} = -2\frac{k\sigma_T}{m_e c^2}\int n_e T_e dl \tag{20.18}$$



SZ Distance Measurements

To derive an SZ distance, one first adopts a model for the distribution (and temperature) of an x-ray gas in a cluster. Recall from our discussion of x-ray clusters, the timescale for equipartion in an x-ray gas is very short ($\sim 10^5$ yr for electrons), so the isothermal assumption should be pretty good. In that case, the radial distribution of the gas can be approximated by

$$n_e(r) = \frac{n_0}{(1+\tilde{r}^2)^{3/2}} \tag{20.19}$$

where $\tilde{r}$ is the radius of the cluster in terms of its core radius, r_0. Now let's choose a line-of-sight through the cluster. (In practice, one integrates over the beam size of the microwave detector, but for simplicity, we'll just use the single path through the middle of the cluster.) The amount of microwave dimuntion is

$$\begin{aligned}\left(\frac{\delta}{T}\right) &= -2\frac{k\sigma_T}{m_e c^2}\int n_e T_e dl \\ &= -2 n_0\, T_e \frac{k\sigma_T}{m_e c^2} \cdot \int_0^\infty \frac{1}{(1+\tilde{r}^2)^{3/2}} r_0 d\tilde{r} \\ &- 4 n_0 T_e \frac{k\sigma_T}{m_e c^2} \cdot r_0 \end{aligned} \tag{20.20}$$

Meanwhile, the x-ray emission from that line-of-sight is

$$\begin{aligned} L_x &= \int n_e^2 \Lambda(T) dl = 2\int_0^\infty \left[\frac{n_0}{(1+\tilde{r}^2)^{3/2}}\right]^2 \Lambda(T) r_0\, d\tilde{r} \\ &= \frac{3\pi}{8} n_0^2\, \Lambda(T) \cdot r_0 \end{aligned} \tag{20.21}$$

Now, if we can resolve the cluster, we can measure the angular size of the core radius. This is related to the linear core radius simply by

$$r_0 = D_A \theta_0 \tag{20.22}$$

Similarly, the x-ray luminosity of the cluster is related to the observed x-ray flux by

$$L_x = D_L^2 l_x = D_A^2 (1+z)^4 l_x \tag{20.23}$$

Note that we have two equations and two unknowns; if we eliminate the central density from the equations, then

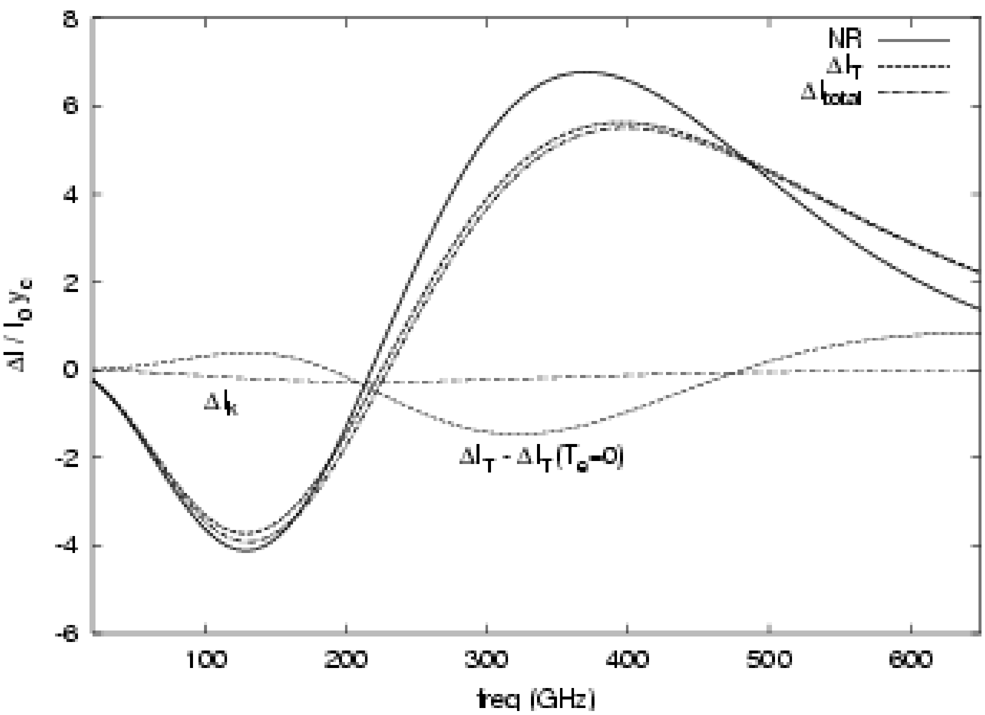
$$D_A = \left\{ \left(\frac{\delta T}{T} \right)^2 \left(\frac{3\pi\Lambda(T)}{128 l_x} \right) \left(\frac{1}{(1+z)^4 T_e^2 \theta_0} \right) \left(\frac{m_e c^2}{k\sigma_T} \right)^2 \right\}^{1/3} \tag{20.24}$$

Thus, we have an equation for the angular size distance that depends only on the two observables: the microwave dimunition and the x-ray flux.

The SZ effect is potentially a powerful tool for measuring distances: since the amount of the microwave dimunition does not depend on redshift, one could, in principle observe extremely distant clusters and derive not only the Hubble Constant, but Λ as well. In practice, however, there are difficulties. The biggest problem lies with the assumption of spherical symmetry. The core radius of SZ clusters comes from observations on the plane of the sky, but the microwave dimunition comes from the radial path through the cluster. If the two are not identical, then an error will result. Note that this will probably not average out over clusters, since x-ray (and optical) surveys preferentially detect clusters that are more elongated along the line-of-sight (thereby increasing the apparent contrast and/or surface brightness of the cluster).

The Kinetic SZ Effect

In addition to measuring distance, the SZ effect can also be used the measure the peculiar motion of clusters with respect to the microwave background. In this case, since the entire cluster is moving in the same direction, the scattering does not distort the blackbody curve; rather it just Doppler shifts it slightly to the red (or blue). However, this effect is about an order of magnitude smaller the the thermal SZ effect (and is impossible to disentagle from a primordial fluctuation without additional information).



The above plot shows the normalized intensity change caused by the thermal and kinetic SZ effects for a 15 keV galaxy cluster with peculiar velocity of 500 km/sec. The solid line is the non-relativistic SZ effect. The dashed lines are the relativistic corrections, both alone and together with the thermal effect. The dot-dashed lines are the kinetic SZ effect, both alone and together with the thermal effect.